MA 339 Jan 3:0

Geometric Analysis

Instructor

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Department: Mathematics

Course Time: Tue., Thu., 2:00-3:30 PM

Lecture venue: LH-5 in the Mathematics department

Detailed Course Page: http://math.iisc.ac.in/all-courses/ma339.html

Announcements

Brief description of the course

This is a graduate course meant for PhD (and integrated PhD) students who want to work in areas like differential geometry, complex analysis, algebraic geometry, topology, and possibly even harmonic analysis. The point is to set up partial differential equations (largely linear elliptic ones) on compact manifolds and study the existence and uniqueness of their solutions. If time permits, then some applications to geometry (like the uniformisation theorem and Poincare duality) will be done.

This course will also have a brief introduction to Riemannian geometry (simply as a language hanging in the backdrop). Some advanced undergraduates in mathematics may also take this course as preparation for graduate study in geometry and topology.

Prerequisites

A first course on manifolds (like MA 338), Analysis (some measure theory, multivariable calculus, function spaces, and basic Fourier analysis), as well as Functional analysis (up to the spectral theorem for compact

self-adjoint operators).

Syllabus

Basics of Riemannian geometry (Metrics, Levi-Civita connection, curvature, Geodesics, Normal coordinates, Riemannian Volume form), The Laplace equation on compact manifolds (Existence, Uniqueness, Sobolev spaces, Schauder estimates), Hodge theory, more general elliptic equations (Fredholmness etc), Uniformization theorem.

Course outcomes

After taking this course, a student ought to

1) Understand what a PDE on a manifold means.

2) Why we care about setting up and studying PDE on manifolds.

3) How one proves existence and uniqueness for linear and some nonlinear PDE.

4) As a part of 3), know about Sobolev spaces and Holder spaces of functions.

Grading policy

25% for homework.

25% for the mid-term.

50% for the final exam.

Assignments

Resources

Do Carmo, Riemannian Geometry.

Griffiths and Harris, Principles of Algebraic Geometry.

S. Donaldson, Lecture Notes for TCC Course - Geometric Analysis.

J. Kazdan, Applications of Partial Differential Equations To Problems in Geometry.

L. Nicolaescu, Lectures on the Geometry of Manifolds.

T. Aubin, Some nonlinear problems in geometry.

C. Evans, Partial differential equations.

Gilbarg and Trudinger, Elliptic partial differential equations of the second order.

G. Szekelyhidi, Extremal Kahler metrics.